MEK 307 Fluid Mechanics

FLUID PROPERTIES

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Dimensions and units (Chapter 1.6)

We use sets of units to properly define, express certain physical properties/phenomenon

Basic dimensions: length [L], mass [M], force [F], time [T]

\[ [F] = [M][LT^{-2}] \]

\[ F = ma \]

SI: International System of Units

\[ N = \text{kg.m/s}^2 \]

BG: British Gravitational System

\[ \text{lb} = \text{Slug.ft/s}^2 \]

\[ \text{Slug} = \text{lb.s}^2/\text{ft} \]
Energy = Force x Distance

<table>
<thead>
<tr>
<th>SI</th>
<th>BG</th>
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<tbody>
<tr>
<td>$= [M][LT^{-2}][L]$</td>
<td>$= [F][L]$</td>
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<tr>
<td>$(kg \cdot m/s^2)(m)$</td>
<td>=lb.ft</td>
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<tr>
<td>$= Nm$</td>
<td>(1.3558 J )</td>
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<td>$= Joule$</td>
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Unit homogeneity:

2+2=?4

Manning’s Equation:

\[ Q = \frac{1}{n} A R^{2/3} S^{1/2} \]

\[ L^3/T = ?[L^2][L]^{2/3} \]

Has units:

\[ [L^{1/3}/T] \]

(1 meter = 3.2808399 feet)

Where:
- \( Q \) = Flow Rate [L^3/T]
- \( A \) = Flow Area, [L^2]
- \( n \) = Manning’s Roughness Coefficient
- \( R \) = Hydraulic Radius, [L]
- \( S \) = Channel Slope, [L/L]

For those who are interested: http://en.wikipedia.org/wiki/Robert_Manning_(engineer)
Properties of Fluids (Chapter 2.2)

Density, \( \rho = \frac{\text{mass}}{\text{unit volume}}, \ [\text{M/L}^3] \)

Specific volume, \( v = \frac{1}{\rho} \), volume occupied by unit mass \([\text{L}^3/\text{M}]\)

Specific weight, \( \gamma = \frac{\text{weight}}{\text{unit volume}}, \ [\text{F/L}^3] \) (depends on latitude)

Specific gravity, \( \text{SG} (s) = \frac{\rho}{\rho_{\text{water}}} \)

http://www.agen.ufl.edu/~chyn

[Graph showing the density of water vs. temperature]

density of fluids vary with temperature
Compressibility of Liquids

\[ E_v = -v \frac{dp}{dv} = -\left( \frac{v}{dv} \right) dp \quad \Rightarrow \quad \frac{v_2 - v_1}{v_1} \approx -\frac{p_2 - p_1}{E_v} \]

\( v = \text{specific volume}, \ p = \text{pressure} \)

Bulk Modulus (volume modulus of elasticity) Units?

We consider liquids to be incompressible but why?
Properties of Fluids

**Fluids**
Deforms continuously or flows when subject to shear force

**Solids**
Can resist a shear force at rest (with some displacement but tend to retain shape)

Shear force: Force component tangent to a surface

Shear Stress: \( \tau = \frac{F_t}{A} \)

Pressure: \( p = \frac{F_n}{A} \)
Properties of Fluids

The strain in a solid is independent of the time over which the force is applied and (if the elastic limit is not reached) the deformation disappears when the force is removed.

A fluid continues to flow for as long as the force is applied and will not recover its original form when the force is removed.
Properties of Fluids

Shear force, $F_t$: Force component tangent to a surface
Normal Force, $F_n$: Force component normal to a surface

Shear Stress: $\tau = \frac{F_t}{A}$
Pressure: $p = \frac{F_n}{A}$

Deformation

Compressibility

Viscosity, $\mu$
Some liquids are more viscous than others

3120 psi needed to compress a unit volume of water 1%
Properties of Fluids (Chapter 2.6)

Viscosity is the means by which the fluid resists friction (shear stress)

\[ \tau = \mu \left( \frac{\partial u}{\partial y} \right) \]

For fluids, the rate of strain (angular deformation) is proportional to the applied stress.

Newtonian Fluids: \( \mu \) is constant

Constant of proportionality, viscosity (dynamic, absolute)
Viscosity (Chapter 2)

For gases $\mu \propto T$

For liquids $\mu \propto T$

$\mu \Rightarrow \text{units?}$
Properties of Fluids

Uniform flow velocity profile

Laminar shear flow velocity profile

In practice we are concerned with flow past solid (sometimes rough) boundaries; pipe walls, river bed etc. and there will always be shear stress.
Viscosity, $\mu$ is a measure of resistance to relative motion of adjacent fluid layers (angular deformation).

$$\tau = \mu \frac{du}{dy}$$
Shear Stress and Viscosity (Chapter 2)

Viscosity, \( \mu \)  \( \rightarrow \)  Fluids resist to shear lightly  \( \rightarrow \)

Ideal fluids  \( \rightarrow \)  Viscosity, \( \mu=0 \)  \( \rightarrow \)  Only normal stresses present

Newtonian fluids  \( \rightarrow \)  Viscosity, \( \mu=\text{const.} \) (for const. temperature)

\[ \frac{du}{dy}=0 \]  \( \rightarrow \)  \( u=u(y)\)=\text{const.}

\( \tau=0 \)

\( u=0, \) fluid is at rest  \( \rightarrow \)  Only normal stresses present

No slip condition: Fluid in the immediate contact with a solid boundary has the same velocity as the solid boundary
Surface Tension (Chapter 2)

Exists whenever there is a density discontinuity
(Interfaces between a gas and a liquid; two different liquids…etc)

Cohesion: Attraction between like molecules, enables a fluid to resist tensile stresses

Adhesion: Attraction between unlike molecules, enables a fluid to stick to another body

http://www.bcscience.com
Vapor Pressure (Chapter 2)

Pressure of a vapor in equilibrium with its non-vapor phases

Vapor pressure depends on temperature

To avoid boiling, pressure on top of liquid should be higher than vapor pressure

Towards the perfect boiled egg on top of Mount Everest

On top of Mount Everest the pressure is about 260 mbar (26.39 kPa, 0.25 atm)

You need water temperature higher than 70°C to cook an egg (for a hard yolk)

Conclusion: you can not boil an egg on top of Mount Everest.
Vapor Pressure, Cavitation (Chapter 2)

Cavitation damage on an impeller of a BW5000 pump
www.lightmypump.com
Fluid Statics (Chapter 3)

Fluid at rest \textbf{(no shear force)}

Normal Force, $F_n$: Force component normal to a surface

($F_n$ : surface force component)

Pressure: $P = \frac{F_n}{A}$
Fluid Statics

Absolute and gage pressures

Absolute pressure = pressure measured relative to absolute zero (perfect vacuum)

Gage pressure = pressure measured relative to local atmospheric pressure

Pressure usually doesn’t affect liquid properties and atmospheric pressure usually appears on both sides of an equation. So we commonly use gage pressure in problems dealing with liquids.
Partial derivative

Differentiating a function of more than one variable with respect to a particular variable, with the other variables kept constant:

the notation $\frac{\partial f}{\partial t}$ means the partial derivative of the function $f$ with respect to $t$

$\frac{\partial f}{\partial t} : \text{partial derivative}$

$\frac{df}{dt} : \text{total derivative}$
Fluid Statics (Chapter 3)

Pressure at a point!

- Pressure at a point is same in all directions.

Variation of pressure in a static Fluid:

- Pressure increases linearly with depth.

\[
\frac{dp}{dz} = -\gamma
\]
Fluid Statics (Chapter 3)

Variation of Pressure in a static Fluid

Pressure increases linearly with depth (for an incompressible fluid)

\[ p = -\gamma z + p_{atm} \]

Pascal’s law: a surface of equal pressure for a liquid at rest is a horizontal plane. It is a surface everywhere normal to the direction of gravity.
Fluid Statics (Chapter 3)

Pressure expressed in height of fluid

\[ h \text{ is the height of fluid at the bottom of which the pressure will be } p \]

Also called *pressure head*

1 Atm = 76 cm Hg

= 10.33 m H₂O

\[ [L] = \frac{[F]}{[L]^2} \times \frac{[F]}{[L]^3} = [L] \]
Fluid Statics

Pressure head, an important property

1) Identify a reference point (datum)

2) Identify elevation of points 1 and 2

3) Sum of elevation of a point and pressure head at that point is constant

\[ h_1 + z_1 = h_2 + z_2 = \text{const.} \]

that is

\[ \frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 = \text{const.} \]
Fluid Statics

Remarks

\[ p = \gamma h \text{ or } h = \frac{p}{\gamma} \text{ when} \]

- Specific weight, \( \gamma = \text{const.} \)
- fluid is at rest
- fluid is incompressible
- the pressure of air or vapor acting on the free surface is neglected (for now)
- distances \((h)\) measured vertically downward from the free surface
Fluid Statics

Pressure head, examples

Which of these pressures is the highest?

\[ p_A = p_B = p_C = p_B = \gamma h \]
Fluid Statics
Pressure head, examples

Which of these points (A, B, C) will experience the highest pressure?

\[ p_A = p_B = p_C = \gamma_{\text{gasoline}} h \]
Fluid Statics

Pressure head, examples

Pressure at the same depth is the same, moving sideways does not change the pressure
Fluid Statics

Pressure head, examples

\[ p_B = ? \]

\[ p_B = \gamma_1 h_1 + \gamma_2 h_2 \]
Fluid Statics

Pressure head, examples

\[ p_2 - p_1 = ? \]

\[ p_1 + \gamma_1(h_1 - h_2) = p_2 + \gamma_1(h_4 - h_3) + \gamma_2(h_3 - h_2) \]

\[ p_2 - p_1 = \gamma_1(h_1 - h_2) - \gamma_2(h_3 - h_2) - \gamma_1(h_4 - h_3) \]
Fluid Statics

Absolute and gage pressures

**Absolute pressure** = pressure measured relative to absolute zero (perfect vacuum)

**Gage pressure** = pressure measured relative to local atmospheric pressure

Throughout the course, unless otherwise specified, pressure will mean gage pressure.
Fluid Statics

Absolute and gage pressures, example

\[ p_B \text{ (gage)} = \gamma_1 h_1 + \gamma_2 h_2 \]

\[ p_B \text{ (abs.)} = \gamma_1 h_1 + \gamma_2 h_2 + p_{\text{atm}} \]

\[ p_B \text{ (abs.)} = p_B \text{ (gage)} + p_{\text{atm}} \]
Fluid Statics

Pascal's Principle

Any external pressure applied to a fluid is transmitted undiminished throughout the liquid and onto the walls of the containing vessel.

\[ P = \frac{f}{a} = \frac{F}{A} \quad \Rightarrow \quad F = \frac{A}{a} f \]

If \( A >> a \) then \( F >> f \)

http://www.ac.wwu.edu/~vawter

This is the basic idea of hydraulic machines!
Fluid Statics

Summary of last lecture

Pressure at a point is the same in all directions

Pressure is constant on a horizontal plane

Pressure increases linearly with depth

\[ \frac{dp}{dz} = -\gamma \Rightarrow \]

liquid with free surface \( p_b = \gamma h \)
Fluid Statics

Pressure head, Example

An open tank contains water 1.4 m deep covered by a 2-m-thick layer of oil ($s = 0.855$). What is the pressure head at the bottom of the tank, in terms of a water column?

\[ p_i = \gamma_o h_o \]

\[ p_i = \gamma_o h_o \]

\[ p_b = p_o + p_w = \gamma_o h_o + \gamma_w h_w \]
Fluid Statics

Measurement of pressure, barometer

\[ p_0 = \gamma y + p_{\text{vapor}} \]

\[ p_a = p_0 \]

\[ p_a = p_{\text{atm}} \]

\[ p_0 = p_a = p_{\text{atm}} = \gamma y + p_{\text{vapor}} \]

\[ p_{\text{atm}} = \gamma y \]
Fluid Statics

Measurement of pressure, other methods

Bourdon gage

Converting energy from the pressure system to displacement of a mechanical system

Pressure transducers

http://www.pchemlabs.com

**Fluid Statics**

**Measurement of pressure, manometer**

**Piezometer column**

\[ p_A = \gamma h + R_M \left( \frac{s_M}{s_F} \right) \gamma + p_{atm} \]

*We can omit portions of the same fluid with the same end elevations (pressure doesn’t change on a horizontal plane).*
Fluid Statics

Measurement of pressure, manometer

Vacuum

\[ p_A = \gamma h - R_M \left( \frac{s_M}{s_F} \right) \gamma + p_{atm} \]
Fluid Statics

Measurement of pressure, manometer

Vacuum

\[ p_A / \gamma_F = -((s_M / s_F)R_m + h) \]
Fluid Statics

Measurement of pressure, differential manometer

Liquid A and B have the same density!

\[
p_A / \gamma - p_B / \gamma = z_B - z_A + R_M - R_M(s_M/s_F)
\]

\[
\Delta \left( \frac{p}{\gamma} + z \right) = \left( 1 - \frac{S_M}{S_F} \right) R_m
\]

\[
p_A / \gamma - h_A - R_M(s_M/s_F) + h_B = p_B / \gamma
\]

\[
p_A / \gamma - p_B / \gamma = h_A - h_B + R_M(s_M/s_F)
\]

\[
\Delta \left( \frac{p}{\gamma} + z \right) = \left( \frac{S_M}{S_F} - 1 \right) R_m
\]
Fluid Statics

Manometer problem

The air pressure in a tank is measured by an oil manometer. For a given oil-level difference between the two columns, determine the absolute pressure in the tank. (density of oil is given to be $\rho = 850$ kg/m$^3$)
Fluid Statics

Differential manometer problem

Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. Determine the pressure difference between the two pipelines.

$h_w = 0.6 \text{ m}, h_{\text{Hg}} = 0.1 \text{ m}, h_{\text{sea}} = 0.4 \text{ m}$

The densities of seawater and mercury are given to be $\rho_{\text{sea}} = 1035 \text{ kg/m}^3$ and $\rho_{\text{Hg}} = 13,600 \text{ kg/m}^3$. The density of water $\rho_w = 1000 \text{ kg/m}^3$. 
Fluid Statics

Piston problem

\[ F_1 = 200 \text{ N}, \quad F_2 = ? \]
Fluid Statics

Measurement of pressure, inclined manometer

Example

The actual vertical rise is still the same as it is in a vertical tube, but here it is much easier to read accurately when the scale has been expanded!

http://www.gnw.co.uk/
Fluid Statics

Example
Fluid Statics

Forces on plane areas, applications: Panama Canal locks

http://sems1.cs.und.edu/~sems/
Fluid Statics

Forces on plane areas, applications: Foundation failure

http://www.helitechonline.com/
Fluid Statics

Forces on plane areas, applications: Gates

Feeder Gates for Canal

Gate Valves for Spillway Control

http://users.owt.com/chubbard/gcdam/html/photos

http://sems1.cs.und.edu/~sems/
Fluid Statics
Forces on plane areas, applications: Dams

Hoover Dam

A beaver dam

http://commons.wikimedia.org/wiki/User:Leoboudv

http://snailstales.blogspot.com
Fluid Statics

Forces on plane areas, horizontal surface

If fluid is at rest:
No tangential forces exist
All forces are normal to the surfaces
If the pressure is uniformly distributed over the area $A$ (e.g. submerged horizontal area for liquids and gases), then:

$$F = \int p \, dA = p \int dA = pA$$

$F$: static fluid force, acts at the centroid of area $A$

If fluid is gas: pressure variation with depth over a surface can be neglected!
Fluid Statics

Forces on plane areas, vertical plane

When the fluid at rest is liquid, then the pressure distribution is not uniform

*F*: static fluid force, **always** acts below the centroid of area \( A \)

The deeper the plane is submerged, the closer the resultant \( F \) moves to the centroid of the surface
Fluid Statics

Forces on plane areas, magnitude of $F$

If the width (into the page), $x$ is constant, then $F = pA = 0.5(p_{MJ} + p_{NK})(MNx)$

If $x$ varies, $dA = xdy$, $p = \gamma h$, $h = y\sin\theta$, then the force $dF$ on the horizontal strip:

$$dF = pdA = \gamma hdA = \gamma y \sin \theta dA$$
Fluid Statics

Forces on plane areas, magnitude of $F$

Integrating

Total force on any plane area (submerged in liquid) can be found by multiplying the specific weight by the product of the area and the depth of its centroid.
Fluid Statics

Forces on plane areas, center of pressure

The point where the line of action of the resultant force goes through is the centroid of the pressure prism (volume).
Fluid Statics

Forces on plane areas, center of pressure

\[ h_p = \frac{2}{3} \text{ MN} \]

Pressure prism concept is useful to determine the center of pressure for simple geometries.
Fluid Statics

Forces on plane areas, center of pressure

If the shape of the area is not regular we have to take moments and integrate:

\[ y_d F = \gamma y^2 \sin \theta dA \]

\[ y_p F = \int_A ypdA = \int_A y(\gamma y \sin \theta) dA = \gamma \sin \theta \int y^2 dA \]
Fluid Statics

Forces on plane areas, center of pressure

\[ ydF = \gamma y^2 \sin \theta \, dA \]

\[ y_p F = \int_A y_p \, dA = \int_A y(\gamma y \sin \theta) \, dA = \gamma \sin \theta \int_A y^2 \, dA \]

\[ I_O = \int_A y^2 \, dA \]

\[ I_O = I_c + y_c^2 A \]

\[ F = \gamma \sin \theta y_c A \]

\[ y_p = \frac{\gamma \sin \theta I_O}{\gamma \sin \theta y_c A} = \frac{I_O}{y_c A} \]
**Fluid Statics**

Forces on plane areas, location of center of pressure

**Parallel axis theorem**

\[ y_p = \frac{I_O}{y_c A} = \frac{Ay_c^2 + I_c}{y_c A} = y_c + \frac{I_c}{y_c A} \]

\( I_c \): moment of inertia of an area about its centroidal axis
Fluid Statics

Forces on plane areas, center of pressure, two methods

1) The point where the line of action of the resultant force goes through is the centroid of the pressure prism (volume).

2) Consider the moments

\[ y_p = y_c + \frac{I_c}{y_c A} \]
Fluid Statics
Forces on plane areas, Steps

1) Determine the area \((A)\) of the submerged surface
2) Find the centroid of the area, \(y_c\), and then \(h_c\)
3) Calculate the magnitude of the resultant hydrostatic force, \(F\)
4) Calculate the line of action of the hydrostatic force, \(y_p\)
Fluid Statics

Forces on plane areas, applications: Gates